

TOPIC 10: BASIC PROBABILITY AND THE HOT HAND

1. THE HOT HAND DEBATE

Let's start with a basic question, much debated in sports circles: "Does the Hot Hand really exist?". A number of studies on this topic can be found in the literature. A summary of the history of the debate may be found [here](#). A seminal paper by Vallone, Gilovich and Tversky, published in 1985, found no evidence of the hot hand in basketball claiming that the belief stems from a tendency for fans and athletes to misjudge the length of streaks that one might expect from randomly generated data [2]. A more recent [study](#) by Bocskocsky, Ezekowitz, and Stein found statistically significant evidence of a hot hand effect in basketball when they adjusted for the level of difficulty of the shots taken by players and the response of the defense when a player was perceived to have the hot hand. A recent [study](#) by Zweibel and Green also finds evidence of the hot hand in baseball.

The interpretation of the question tends to vary widely; the definition of the hot hand, whether evidence for it should be observable in performance statistics or whether we should also incorporate changes in factors influencing performance in our observations, and whether we are asking the question about all players or just one should all be agreed upon before exploring the question itself. Rather than give an answer of yes or no to the question, we will help you develop some methods of exploration of the question through basic probability.

Our interpretation of the basic assumption of the "hot hand" will be that the probability that a player will succeed at a task (which is repeated) is higher if the task has resulted in success rather than failure on the previous try. Sometimes the hypothesis is that the probability of success is higher than normal after a string of success of some specified length or after a recent increase in statistics measuring performance. The tools developed below will be equally applicable to these hypotheses. The question quickly brings us to the heart of the meaning of probability and opens our eyes to some of the common misjudgments in our expectations. We will explore the question of what we should expect from data generated randomly and learn how to decide if an individual player's performance deviates significantly from that expectation.

2. EXPERIMENTS AND SAMPLE SPACES

Almost everybody has used some conscious or subconscious estimate of the likelihood of an event happening at some point in their life. Such estimates are often based on the relative frequency of the occurrence of the event in similar circumstances in the past and some are based on logical deduction. In this section we will set up a framework within which we can assign a measure of the likelihood of an event occurring (probability) in a way that reflects our intuition and adds clarity to help us with our calculations.

Definition 2.1. *An **Experiment** is an activity or phenomenon under consideration. The experiment can produce a variety of observable results called **outcomes**. The theory of probability makes most sense in the context of activities that can be repeated or phenomena that can be observed a number of times. We call each observation or repetition of the experiment a **trial**.*

Examples

- (1) Flip a fair coin and observe the image on uppermost face (heads or tails).
- (2) Flip a fair coin three times and observe the resulting sequence of heads (H) and tails(T).
- (3) Rolling a fair six sided die and observing the number on the uppermost face is an experiment with six possible outcomes; 1, 2, 3, 4, 5 and 6.
- (4) Take a penalty shot in soccer and observe whether it results in a goal or not.
- (5) Rolling a six sided die and observing whether the number on the uppermost face is even or odd is an experiment with two possible outcomes; "even" and "odd".

- (6) Pull a marble from a bag containing 2 red marbles and 8 blue marbles and observe the color of the marble.

Definition 2.2. *A sample space for an experiment is the set of all possible outcomes of the experiment. Elements of the sample space are sometimes simply called **outcomes** or if there is a risk of confusion they may be called a **sample points** or **simple outcomes**.*

In our above examples the sample spaces are given by

Experiment	Sample Space
Flip a fair coin	{Heads, Tails }
Flip a fair coin	{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Roll a six-sided die and observe number on uppermost face	{1, 2, 3, 4, 5, 6 }
Take a penalty shot	{ Goal, No Goal }
Roll a six-sided die and observe whether the number is even or odd	{ Even, Odd }
Pull a marble from a bag containing 2 red marbles and 8 blue marbles and observe the color of the marble.	{Red, Blue }

Note that the sample space depends not just on the activity but also on what we agree to observe. In cases where there is any likelihood of confusion it is best to specify what the observations should be in order to determine the sample space.

When specifying the elements of the sample space, S for an experiment, we should make sure it has the following properties:

- (1) Each element in the sample space is a possible outcome of the experiment.
- (2) The list of outcomes in the sample space covers all possible outcomes of the experiment.
- (3) No two outcomes in the sample space can occur on the same trial of the experiment.

3. ASSIGNING PROBABILITIES TO OUTCOMES IN A SAMPLE SPACE

For most experiments, the outcomes of the experiment are unpredictable and occur **randomly**. The outcomes cannot be determined in advance and do not follow a set pattern. Although we may not be able to predict the outcome on the next trial of an experiment, we can sometimes make a prediction about the proportion of times an outcome will occur if we run “many” trials of the experiment. **We can make this prediction using logic or past experience** and we use it as a measure of the likelihood (**the probability**) that the outcome will occur.

Example 3.1. *If I flip a fair coin 1,000 times, I would expect to get roughly 500 heads and 500 tails, in other words, I would expect half of the outcomes to be heads and half to be tails. This prediction is based on logical deduction from the symmetry of the coin. In this case I would assign a probability of $1/2$ to the outcome of getting a head on any given trial of the experiment.*

Example 3.2. *If I roll a six sided precision cut die 6,000 times, I would expect to get a six on the uppermost face about 1,000 times. This prediction is based on the logical deduction that all faces of the die are equally likely to appear on the uppermost face when the die is thrown because of the symmetry of the die. In this case, I would assign a probability of $1/6$ to the outcome “6” on any given trial of the experiment.*

Example 3.3. *I could assign a measure of 0.496 to the likelihood or probability that **LeBron James** will make the next field goal he attempts in a regular season game, based on his current career field goal percentage of 49.6% (on Feb 07 2015; click on the link to find his current FG%).*

3.1. Reasoned vs. Empirical Probability. When we observe a number of trials of an experiment and record the frequency of each possible outcome, **the relative frequency of an outcome is the proportion of times that it occurs**. When we use the relative frequency as a measure of probability we are using **empirical evidence** to estimate the likelihood of an event. The **empirical probability** calculated in this

way **may not agree with our reasoned probability** (see the law of large numbers below). Our measure of the probability that LeBron James will make the next field goal he attempts is an example of such an empirical estimate.

Activity 1: Can we guess what's in the bag? We will have Student X leave the room. We will place 10 marbles in a bag. The marbles will be blue and yellow and we will know how many of each are in the bag. Thus we will be able to use logical reasoning to deduce the probability of drawing a marble of a particular color from the bag if that marble is drawn at random.

Student X will now return to the room and draw a marble at random from the bag, record its color and return it to the bag. Student X will repeat the process 10 times and use the empirical evidence to guess what proportion of marbles in the bag are blue and what proportion are yellow.

Hopefully the above activity has been useful in demonstrating that the two different methods of assigning probabilities to the outcomes in a sample space, using logical deduction and using relative frequency, can result in different probability assignments. A lack of understanding of this discrepancy often causes confusion. However, if we use a large number of trials to determine relative frequencies, the discrepancy is likely to be small.

The Law of large numbers says that if an experiment is repeated "many" times, the relative frequency obtained for each outcome approaches the actual probability of that outcome.

This means that if we have used (sound) logic to arrive at a probability for an outcome and if we run our experiment "many" times, the relative frequency of the outcome should be "close" to our reasoned probability. This of course raises many questions about the number of trials required to get a good estimate etc... One needs to explore statistics in more detail in order to answer these questions thoroughly. For now we will satisfy ourselves with the knowledge that both of our methods of calculating probability agree.

Example 3.4. *The law of large numbers says that if student X above were to repeat the process of drawing marbles from the bag and replacing them 1,000 times, we would expect the relative frequency of each color to reflect the proportion of marbles in the bag that are of that color.*

3.2. Simulating Experiments. We can use the following applet developed at The University of Alabama in Huntsville to explore relative frequencies and later to explore the sequences of outcomes resulting from many trials of the same experiment.

A Useful Simulation Open the applet [Flipping a Coin](#). To simulate flipping a coin 10 times, set $n = 1$ to indicate that you are flipping a single coin, set $p = .5$ to indicate that it is a fair coin, set $Y = \text{Number of heads}$ and set the "Stop" button at .

Activity 4: How do runs in the data generated randomly compare with our expectations?

Lets assume that a basketball player has a fifty percent chance of getting a basket each times he/she shoots and if he/she shoots 10 times in a row. Repeat the procedure in part (b) but this time record the sequence of baskets and misses below for each simulation, with 1 representing a basket and a 0 representing a missed shot. Note the variation in the pattern of runs or streaks of baskets and misses in the data. This should demonstrate that streaks of success do not necessarily indicate that a player has a “hot hand”, they may just be due to random variation.

Simulation #	1	2	3	4	5	6	7	8	9	10
Trial 1: Y value										
Trial 2: Y value										
Trial 3: Y value										
Trial 4: Y value										
Trial 5: Y value										
Trial 6: Y value										
Trial 7: Y value										
Trial 8: Y value										
Trial 9: Y value										
Trial 10: Y value										

Now that we have discussed the two methods of assigning probabilities to outcomes, lets get back to the details of doing so and the methods of presenting those probabilities.

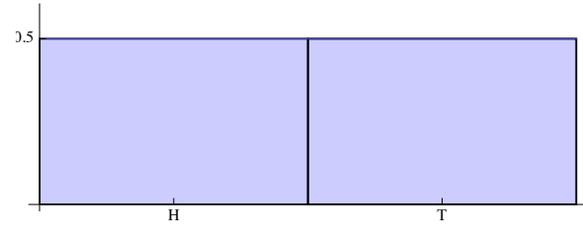
3.3. Rules for assigning probabilities and probability distributions. Let $S = \{e_1, e_2, \dots, e_n\}$ is the sample space for an experiment, where e_1, e_2, \dots, e_n are the simple outcomes. To each outcome, $e_i, 1 \leq i \leq n$, in the sample space, we assign a probability which we denote by $P(e_i)$. As discussed above, the probability assigned to an outcome should reflect the relative frequency with which that outcome should occur in many trials of the experiment. By the law of large numbers, probabilities assigned as a result of logical reasoning should be almost identical to those derived from data for a large number of trials of the experiment. Since the relative frequency of an outcome in a set of data is the proportion of the data corresponding to that outcome, the sum of the relative frequencies of all outcomes must add to 1. With this in mind, we respect the following **basic rules** when assigning probabilities to outcomes in a finite sample space $S = \{e_1, e_2, \dots, e_n\}$:

- (1) $0 \leq P(e_i) \leq 1, \quad 1 \leq i \leq n$
- (2) $P(e_1) + P(e_2) + \dots + P(e_n) = 1.$

We can present our list of the probabilities (called the **probability distribution**) in table format or as a bar chart. In the table format, the probabilities are listed alongside the corresponding outcome in the sample space. When displaying probabilities on a bar chart, we list the outcomes on the horizontal axis and center a rectangular bar above each outcome, with height equal to the probability of that outcome. By making each bar of equal width, we ensure that the **graph follows the area principle** in that the proportion of the area of the graph devoted to each outcome is is equal to its probability.

Example 3.5. *If we flip a fair coin and observe whether we gat a head or a tail, then using logic, we assign a probability of 1/2 to heads and a probability of 1/2 to tails. We can represent this in a table or on a graph as shown below:*

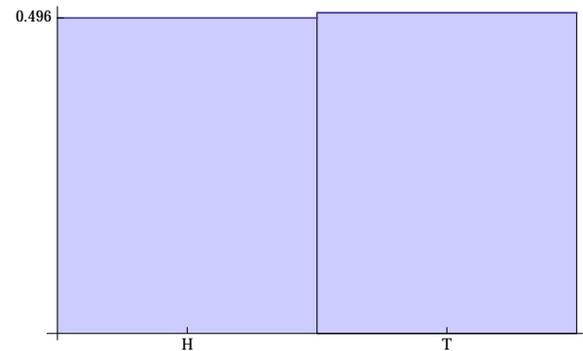
Outcome	Probability
H	$1/2$
T	$1/2$



Example 3.6. If our experiment is to observe LeBron James take his next shot in a basketball game, we might use the empirical evidence we have (an overall field goal percentage of 49.6%) to estimate his probability of success. This gives us the following probability distribution:

LeBron James

Outcome	Probability
<i>Basket</i>	0.496
<i>Miss</i>	0.504

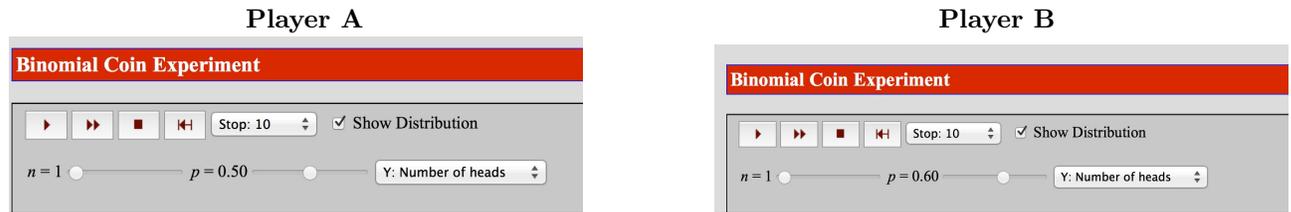


Example 3.7. If our experiment is to observe Amir Johnson take his next shot in a basketball game, we could use his career field goal percentage (or his field goal percentage over the last ten games) to estimate his probability of success on his next shot. We can find the appropriate statistics here: [Amir Johnson](#)

Amir Johnson

Outcome	Probability
<i>Basket</i>	
<i>Miss</i>	

Activity 5: Can we perceive the difference between two players of differing abilities by comparing their performance on ten shots in a row? To explore this we will work in pairs. We will assume that we have two basketball players, Player A with a probability of success equal to 0.5 on each shot and Player B with a probability of success equal to 0.6 on each shot. We will model the situation where each player takes 10 shots in a row where their probability of success remains constant on each shot. One student (let's call him/her Student 1 to be creative) will (secretly) model ten throws by Player A and Player B on our applet [Flipping a Coin](#) using the settings shown below:



Student 1 will write down both sequences of baskets (an outcome of $Y = 1$ gives a basket) and misses ($Y = 0$) and present them to the other student, Student 2. Student 2 will then guess which player generated which sequence of outcomes and after they have guessed Student 1 will reveal which sequence belongs to which player.

You should repeat the process 5 times to see how many times the guess is accurate out of 5.

Note You are likely to observe that this model does not take into account the levels of difficulty of shots that players might take throughout the course of a game. We will talk about this issue later.

This should confirm the difficulty of perceiving differences in ability (and probability) from small data samples and make us wary of jumping to conclusions from short term information.

Pitfalls to avoid when making predictions with probability

- When estimating the overall probability of an outcome with relative frequencies, it is best to avoid using small data sets.
 - If the probability of one outcome is larger than another, it does not guarantee that the more probable outcome will occur on the next trial of the experiment, rather it means that in over many trials of the experiment, the more likely outcome will have a greater relative frequency.
 - It is very difficult to determine an athlete's ability or success rate from a small sample of data. In particular it is difficult to determine if a player's success rate has changed from short term data.
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3.4. Equally Likely Outcomes. Now let's get **back to the hot hand question**. Note that the question bases the prediction for future success on the fact that a run of successes has just occurred as opposed to just an abnormally large proportion of success'. To explore the question of the hot hand further, we need (for the purposes of comparison) to get some idea of the expected frequency of various lengths of runs of success' that should be expected in data generated randomly. We start our exploration by using reasoning

to derive the probability of runs of different length. Let's start with the simple experiment of flipping a coin several times.

Example 3.8. *We first assign probabilities to the outcomes of an experiment which we have neglected. We saw that if we flip a fair coin three times and observe the sequence of heads and tails, we get 8 outcomes in our sample space:*

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

To assign probabilities to each outcome, we reason that we would expect all of these outcomes to be equally likely because of the symmetry of the coin. Thus since all 8 outcomes get equal probabilities and those probabilities must add to 1, each outcome should be assigned a probability of $1/8$. Thus our probability table or distribution is the one shown on the right.

Outcome	Probability
<i>HHH</i>	$1/8$
<i>HHT</i>	$1/8$
<i>HTH</i>	$1/8$
<i>HTT</i>	$1/8$
<i>THH</i>	$1/8$
<i>THT</i>	$1/8$
<i>TTH</i>	$1/8$
<i>TTT</i>	$1/8$

We pull out this general principle for future use:

Equally Likely Outcomes If an experiment has N outcomes in its sample space and all of these outcomes are equally likely, then each outcome has a probability of $1/N$.

The above reasoning extends to flipping a coin any number of times.

Example 3.9. *If we flip a coin 6 times, we get $2^6 = 64$ equally likely outcomes.*

We can see this as follows. If we were to construct a sequence of heads and tails of length 6, we could do so in 6 steps:

Step 1 choose the first letter in the sequence, we have two choices H and T and thus two possible starts to the sequence

H _ _ _ _ _

or

T _ _ _ _ _

Step 2: Given that we have already decided on the first letter, we choose a second letter, giving us 2 choices for each of the above possibilities and thus $2^2 =$ four possibilities for what the sequence might look like after two steps:

H H _ _ _ _

H T _ _ _ _

T H _ _ _ _

T T _ _ _ _

Step 3: Choose the third letter; we see that at the end of this step, we have double the possibilities from the end of the previous step, since we can insert either H or T in the third slot for each. Thus we have $2^3 = 8$ possible starts to the sequence after three steps.

<u>H</u>	<u>H</u>	<u>H</u>	—	—	—	<u>H</u>	<u>H</u>	<u>T</u>	—	—	—
<u>H</u>	<u>T</u>	<u>H</u>	—	—	—	<u>H</u>	<u>T</u>	<u>T</u>	—	—	—
<u>T</u>	<u>H</u>	<u>H</u>	—	—	—	<u>T</u>	<u>H</u>	<u>T</u>	—	—	—
<u>T</u>	<u>T</u>	<u>H</u>	—	—	—	<u>T</u>	<u>T</u>	<u>T</u>	—	—	—

The pattern continues thus with the number of possibilities doubling on each step. By the end of the sixth step, we have completed the sequence and there are $2^6 = 64$ possible outcomes. Since all of these sequences are equally likely and they account for all outcomes of the sample space, we assign a probability of $1/64$ to each one.

In particular, this means that the probability of getting a sequence consisting entirely of heads or a run of eight heads is $HHHHHHH$ is $1/64$ as is the probability of getting a sequence consisting entirely of tails $TTTTTTT$ or the probability of getting a sequence of the form $HHHHHT$.

By generalizing the above analysis to an experiment where we flip a coin N times, we see:

If I flip a fair coin N times, the probability of getting N heads is $1/2^N$. Likewise the probability of getting a sequence of $N - 1$ heads followed by a tail is $1/2^N$.

Activity 6: Before you go any further, make up and write down a sequence of Heads (H) and Tails (T) that you think might result from flipping a coin 100 times in the space below. We will check later how well it matches what we might expect in a sequence that is generated randomly.

3.5. Modelling runs of success' for an athlete with a 50% chance of success. We would like to get a handle on roughly **how many runs of successes of a given size to expect in a large set of data generated randomly.** This should help us to detect unusual patterns in athletic data by comparison. Our random model is still limited to situations where the probability of success is equal to the probability of failure, we will address this issue after we consider conditional probability in the next section.

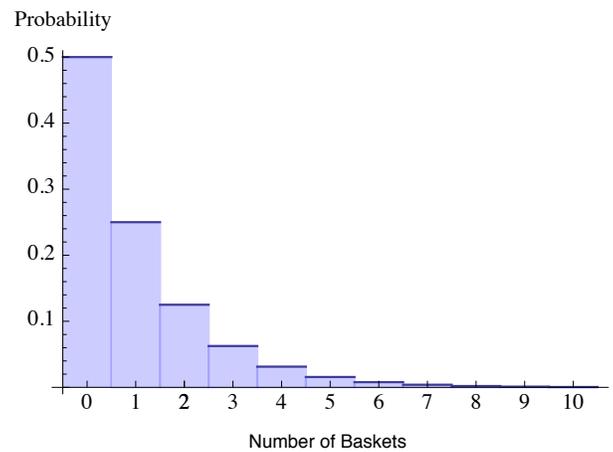
Consider a basketball player who has a 50% chance of making a basket each time they take a shot. Over the course of a season, a major league basketball player might take 800 shots. Instead of thinking of this as one big experiment of 800 shots in a row, we will look at it from a different perspective and think of the player as repeating the following experiment many times:

Experiment A The player shoots until the first time he/she misses a shot and then the experiment is over. The outcomes observed from this experiment will be all possible strings of Baskets (B) followed by a single Miss (M). Thus the sample space looks like:

$$\{M, BM, BBM, BBBM, BBBBM, BBBBBM, \dots\}$$

The “...” symbol in mathematics translates to “etcetera” in english. Notice that this is an infinite sample space; it is conceivable that the unbroken run of baskets could go on forever. Because of our analysis above on coin flipping, we can sign a probability to each of these outcomes. The probability of getting N baskets followed by a miss is $\frac{1}{2^{N+1}}$. The **probability distribution** for this experiment is shown below:

Length of run of Baskets	Outcome	Probability
0	M	1/2
1	BM	1/4
2	BBM	1/8
3	BBBM	1/16
⋮	⋮	⋮
N	$\underbrace{BB \dots B}_N M$	$\frac{1}{2^{N+1}}$
⋮	⋮	⋮



We now model the string of baskets and misses that a basketball player with a 50% of success might generate throughout the season assuming that the player maintained a constant 50% chance of success on each shot. Instead of looking at this as repeated trials of the experiment where we flip a coin once (which gives no information of the length of runs we might expect), we look at the player’s sequence of shots as repeated trials of experiment A, whose probability distribution gives us some idea of how frequently we might expect runs of various lengths to occur. The resulting string of baskets and misses for the player is obtained by concatenating the results of the consecutive trials of the above experiment.

Example 3.10. *If a player ran five consecutive trials of this experiment, getting runs of baskets of length 3, 2, 0, 0, 1 in that order, then the resulting string of baskets and misses for the player would be BBBMBBM-MMBM. Note that the player has taken 11 shots at this point and the number of trials of the experiment is not equal to the number of shots taken. In fact this would only be true if the player missed all of his/her shots.*

What can we expect in a large number of trials? Recall the Law of Large Numbers above that says our reasoned probability of an outcome should be close to the the relative frequency of the outcome in a large number of trials of the experiment. Thus if a player repeated this experiment 400 times, we would expect about 1/2 of those trials (200) to result in a run of baskets of length 0 (M). We would expect about 1/4 of the trials (100) to result in a run of baskets of length 1 (BM) etc.... .

Length of run of Baskets	Outcome	Expected Number
0	M	$\frac{1}{2} \times 400 = 200$
1	BM	$\frac{1}{4} \times 400 = 100$
2	BBM	$\frac{1}{8} \times 400 = 50$
3	BBBM	$\frac{1}{16} \times 400 = 25$
\vdots	\vdots	\vdots
N	$\underbrace{BB \dots B}_N M$ N times	$\frac{1}{2^{N+1}} \times 400$
\vdots	\vdots	\vdots

Note that the number of runs of baskets of any given type has to be a whole number; 0, 1, 2, 3, If the expected number of runs of baskets of a given type turns out to be something other than a whole number, we might adjust the expected number by rounding off to the nearest whole number.

Notice that for a run of 9 baskets (BBBBBBBBBM), the expected number of outcomes of this type in 400 trials is $400 \times \frac{1}{2^{10}} \approx 0.3906$ which is less than 1 (the symbol “ \approx ” in mathematics translates to the word “approximately” in english). In this case, we might adjust our expectations to expect no runs of length 9.

The Longest Run Note that if a player repeated Experiment A 400 times, we expect that about half of the 400 outcomes to have 1 basket or more (since roughly half of the outcomes should be basket runs of length 0 (M)). We expect about a quarter of the 400 outcomes to have 2 baskets or more, about 1/8 of the 400 outcomes to have 3 baskets or more ... about $1/2^N$ of the 400 outcomes to have N baskets or more. At some point, we will find a final value of N for which $\frac{1}{2^N} \times 400$ rounds to 1, indicating that we would expect only one run of length N or greater in the data. This gives us a rough estimate that the longest run of baskets should have length N roughly. By trial and error, we see that

$$0.78 \approx \frac{400}{2^9} \text{ and } 0.39 \approx \frac{400}{2^{10}}$$

Rounding off, we see that **he longest run in 400 trials of Experiment A should have length roughly equal to 9.**

(Those who know a little about logarithms will see that the longest run should have length approximately equal to L , where L is the solution to the equation

$$\frac{1}{2^L} \times 400 = 1$$

The solution to this equation is given by $L = \log_2 400$, we can round off to the nearest integer to get the expected length of the longest run.)

The longest run in M trials of Experiment A Similarly, if we run Experiment A R times, we expect the longest run to have length approximately equal to $\log_2 R$ or the largest integer L for which $\frac{1}{2^L} \times R$ rounds off to 1.

Activity 7: Comparing Data to Expectations

Randomly generated data Lets look at some data generated randomly to see how it compares with our predictions. The following string of data was generated by the Excel file Trials.xlsx. We used Excel to simulate 50 trials of this experiment which resulted in 86 shots. In this case, we would expect about twenty five runs of baskets of length 0, twelve runs of baskets of length 1, six runs of baskets of length 2, three runs of baskets of length 3, one run of baskets of length 4 and none higher than that. Keep in mind however that the law of large numbers just gives us a rough idea of what to expect and in this case we are looking at a relatively low number of trials. Also, just because the chances of a string of 10 baskets in a row are

slim, we cannot rule it out, any outcome with a positive probability can happen, no matter how small that probability is.

The data output is shown below with the trial number on the left and the outcome on the right. Count the number of runs of Bastes of each type and check how well it agrees with our predictions. (Note a run of baskets of length 0 corresponds to a trial resulting in a single miss).

1 M	11 B	22 M	35 M	49 M
2 B	11 B	23 B	36 M	50 M
2 B	11 M	23 B	37 M	
2 M	12 M	23 M	38 M	
3 B	13 M	24 M	39 B	
3 B	14 B	25 B	39 M	
3 M	14 B	25 M	40 M	
4 M	14 M	26 B	41 B	
5 B	15 M	26 B	41 B	
5 M	16 M	26 M	41 M	
6 B	17 B	26 M	42 M	
6 M	17 M	27 M	43 B	
7 B	18 M	28 M	43 B	
7 B	19 B	29 B	43 B	
7 M	19 B	29 M	43 B	
8 B	19 M	30 M	43 M	
8 M	20 M	31 M	44 M	
9 B	21 B	32 B	45 M	
9 M	21 M	32 B	46 M	
10 B		32 M	47 M	
10 B		33 M	48 M	
10 M		34 M		

(b) Check the sequence of heads and tails that you made up above in the Activity 6. How well do the number of runs of baskets of each type compare with what you would expect in a randomly generated sequence.

3.6. What to expect when we flip a coin K times. Notice that when the basketball player (with a 50% chance of making every shot) performs R trials of Experiment A above, he/she usually ends up with a lot more than R shots. We saw that 50 trials of Experiment A led to 74 shots in the randomly generated sample in Activity 7. In fact R trials of Experiment A would only lead to R coin flips if every shot was a miss. Since each trial of Experiment A ends with a miss, the number of completed trials of Experiment A in a set of data is equal to the number of Misses in the data.

If we flip a coin K times, where K is very large, we would expect (by the law of large numbers) that roughly $K/2$ of the outcomes would be Tails. Likewise, if a player with a 50% chance of making a basket on every shot shoots K times where K is large, roughly $K/2$ of the shots will result in Misses. Therefore the player will have run roughly $K/2$ trials of Experiment A.

Below we show the distribution of runs of baskets we should expect from the experiment: “Flip a coin K times”.

Experiment: Flip a coin K times

Length of run of Baskets	Outcome	Expected Number
1	B	$\frac{1}{4} \times \frac{K}{2}$
2	BB	$\frac{1}{8} \times \frac{K}{2}$
3	BBB	$\frac{1}{16} \times \frac{K}{2}$
\vdots	\vdots	\vdots
N	$\underbrace{BB \dots B}_{N \text{ times}}$	$\frac{1}{2^{N+1}} \times \frac{K}{2}$
\vdots	\vdots	\vdots

Example 3.11. *If a player with a 50% chance of success on every shot takes 100 shots in a row, how many runs of baskets of length 4 would you expect to see in the data?*

The longest run of heads in K flips of a coin: (see [1] for further development of this topic.) If a coin is flipped K times, we would expect the length of the longest run to be around $\log_2(\frac{K}{2})$ or the largest value of L for which $\frac{1}{2^L}(\frac{K}{2})$ rounds to a whole number bigger than 0.

Example 3.12. *If a player with a 50% chance of success on every shot takes 100 shots in a row, what is the longest run of data (roughly) that you expect to see in the data?*

Example 3.13. *In the above example of 128 shots taken by LeBron James, what is the longest run of baskets we might expect (using the fact that the overall FG% for LeBron James was approximately 0.5 at the time of these games)? What is the longest run of baskets in the data?*

3.6.1. *What about Tails?* In the above discussion, our focus was on the length of runs of heads over the course of several flips of a coin. A little reflection on what we have done will show that since heads and tails are equally likely on each coin flip, we can expect exactly the same distribution of runs of tails over the course of several coin flips as we did by heads; that is we expect $\frac{1}{2^{N+1}} \times \frac{K}{2}$ runs of tails of length N if we flip a coin K times and we expect the longest run to be of length close to $\log_2(\frac{K}{2})$.

Example 3.14. *In the above example of 128 shots taken by LeBron James, what is the longest run of misses we might expect (using the fact that the overall FG% for LeBron James was approximately 0.5 at the time of these games)? What is the longest run of misses in the data?*

REFERENCES

1. Mark F. Schilling, *The surprising predictability of long runs*, Math. Mag. **85** (2012), no. 2, 141–149. MR 2910308
2. R. Vallone T. Gilovich and A. Tversky, *The hot hand in basketball: on the misinterpretation of random sequences*, Cognitive Psychology **17** (1985), no. 3, 295–314.